Problem 1

Consider an arbitrary symmetric, two-player, two-strategy game. How many Nash equilibria could the game have? For each number you list give an example of a game with that number of Nash equilibria. Prove that you have provided an exhaustive list of possibilities. What about ESS?

Problem 2

In class I said that an ESS is stable with respect to "sufficiently small" mutations. What if the mutations were insufficiently small? Either provide an example or prove that you cannot for the following two statements:

1. A strategy s such that for all other strategies s', $u(s,s) > u(s',s)$ but where s' could invade with a sufficiently large number of mutants
2. A strategy s that is an ESS, but where there is a strategy s' such that $u(s,s) = u(s',s)$ where s' could invade with a sufficiently large number of mutants

Problem 3

I gave two definitions of ESS in class:

Definition 1. *A strategy s is an Evolutionarily Stable Strategy if and only if*

1. *$u(s,s) \geq u(s,s')$ for all $s' \in S_i$* ***and***
2. *If $u(s,s) = u(s,s')$ then $u(s,s') > u(s',s')$*

Definition 2. *A strategy s is an ESS iff there exists a neighborhood N around s such that $p \in N$ implies $u(s,p) > u(p,p)$*

Prove they are equivalent.

Problem 4

Remember the game from class:

	A	B	C
A	0	2	0
B	2	0	0
C	1	1	0

Prove that the set $S = \{(1/2)xA + (1/2)xB + (1-x)C | 1 \geq x \geq 0\}$ is an ES Set.