This homework is going to deal with a new dynamic we haven't studied in class. This is like the best response dynamics, except it takes into consideration how much better or worse a response is. Things that are better responses, even if not the best response, can grow in the population.

First, we must define a weight for a strategy in a population:

$$w_i(p) = \max\{0, u(i,p) - u(p,p)\}$$

Now, the dynamics:

$$\dot{p_i} = w_i(p) - p_i \sum_j w_j(p)$$

The two population case is defined similarly. The weight for a strategy i for player's 1 and 2:

$$w_i^1(p,q) = \max\{0, u_1(i,q) - u_1(p,q)\}$$
$$w_i^2(p,q) = \max\{0, u_2(i,p) - u_2(q,p)\}$$

and the dynamics:

$$\dot{p_i} = w_i^1(p,q) - p_i \sum_j w_j^1(p,q)$$
$$\dot{q_i} = w_i^2(p,q) - q_i \sum_j w_j^2(p,q)$$

Problem 1

Can you characterize the set of fixed points for both the one-population and two-population cases for some arbitrary finite game G?

Problem 2

Consider the three non-degenerate cases for symmetric 2x2 games (the Prisoner's Dilemma class, the Coordination class, and the Hawk-Dove class). How does the one population dynamics behave for this class? Prove what you claim.

Problem 3

Now consider how the two-population version of this dynamics behaves for the three classes of symmetric games we just considered. Can you prove how it behaves for those games?