Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

In sealed bid, second price auctions every individual submits just one bid without knowing what the bids of the other people are. The winner is the person who submits the highest bid, but she only has to pay the value of the bid of the *second* highest bidder. So if the bids are \$10, \$8, and \$5. The person who bid \$10 pays \$8 for the item.

Suppose that each individual has a distinct value for the item, represented by v_i where i represents the bidder. Each individual chooses a bid, b_i for the item. If they win they get the item which is equivalent to them getting v_i, but they have to pay the second highest bid so their payoff is the difference between v_i and the amount they paid. Everyone else gets nothing and pays nothing, so their payoffs are zero.

What strategies survive iterative deletion of weakly dominated strategies? Does the iterative deletion of dominated strategies remove any Nash equilibria? If so, illustrate one that has been removed.

Problem 2

Consider a variant on the ultimatum game. Carlos and Shannon will split a dollar. Carlos moves first and proposes a split to Shannon (he chooses $x \in [0, 1]$). Shannon observes Carlos' offer and has two options: she can accept in which case she gets $(1 - x)$ and Carlos gets x or she can reject. If she rejects she makes a counterproposal to Carlos, but because they are taking too long the pie shrinks (formally, Shannon chooses a $y \in [0, \delta]$ where $0 < \delta < 1$). Carlos can then either accept Shannon's counterproposal in which case Carlos gets $(\delta - y)$ and Shannon gets y or he can reject. If Carlos rejects they both get 0.

What are the subgame perfect equilibria of this game? (You may make a few simplifying assumptions about what Carlos and Shannon do when indifferent or the general properties of their strategies if you think it might help.)

Extra credit

Suppose this process continues indefinitely. Carlos if he rejects can make a counterproposal in $[0, \delta^2]$ and then Shannon can make a counter proposal in $[0.\delta^3]$, etc. What are the subgame perfect equilibria here?

Problem 3

Recall the game of chicken (here the strategies are named "hawk" and "dove"):

	H	D
H	0, 0	3, 1
D	1, 3	2, 2

Consider the indefinitely repeated game of chicken with discount factor δ. Strategy **ALT-H** alternates between H and D every round (starting with H) so long as the other player does the opposite action. If a player deviates and fails to do the opposite action, then **ALT-H** plays H forever. Strategy **ALT-D** is exactly the same, except it starts with D and alternatives D and H.

For what values of δ is the pair (**ALT-H**, **ALT-D**) a Nash equilibrium in the repeated game? Show how you got your answer. How does this equilibrium compare to the three Nash equilibria in the single shot game?

Problem 4

	A	B	C
A	0,0	3,1	0,0
B	1,3	0,0	0,0
C	0,0	0,0	1,1

Find all the Nash equilibria of this game. Which of these Nash are ESS? Can you predict what the evolutionary dynamics of this game would look like (drawing a picture is fine)?