Problem 1

Part A

Find all the pure strategy Nash equilibria for the following game:

	H	T
h	$3,7$	$4,4$
t	$4,3$	$0,5$

Part B

Find all the mixed strategy Nash equilibria.

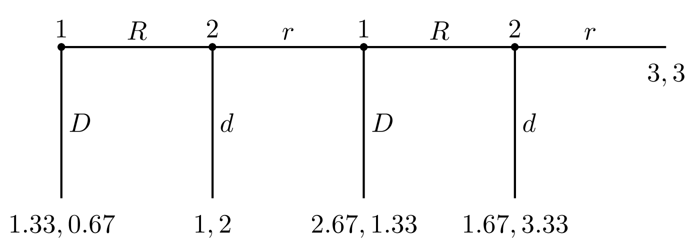
Problem 2

In Santa Fe there is a bar named El Farol. On a given night suppose that three groups of friends are considering going to El Farol (treat each group like it is a single player). El Farol is a cool bar, but it's a little too small to hold all three groups. If a group goes to El Farol and there are two or fewer groups there, the payoff from going is 2. If however all three go the payoff is -1/2. The utility for staying home for each group is 0.

What are the (pure and mixed) Nash equilibria in this game? Which equilibria are *socially optimal* – that is they maximize the sum of the utilities? Is there anything that makes the socially inferior Nash equilibria appealing?

Problem 3

Consider the centipede game that I mentioned in the lecture



Part A What is the normal form representation of this game?

Part B What are the pure strategy Nash equilibria of this game?

Part C Which Nash equilibria are subgame perfect?

Problem 4

Consider a variant on the ultimatum game. Carlos and Shannon will split a dollar. Carlos moves first and proposes a split to Shannon (he chooses $x \in [0, 1]$ which he will keep leaving Shannon the rest). Shannon observes Carlos' offer and has two options: she can accept in which case she gets $(1 - x)$ and Carlos gets x or she can reject. If she rejects she makes a counter proposal to Carlos, but because they are taking too long the pie shrinks (formally, Shannon chooses a $y \in [0, \delta]$ where $0 < \delta < 1$). Carlos can then either accept Shannon's counter proposal in which case Carlos gets $(\delta - y)$ and Shannon gets y or he can reject. If Carlos rejects they both get 0.

What are the subgame perfect equilibria of this game? (You may make a few simplifying assumptions about what Carlos and Shannon do when indifferent or the general properties of their strategies if you think it might help.)

Difficult

Suppose there is another round after this. Now Carlos can make a counter-counter proposal If Carlos rejects can make a counter-counter proposal in $[0, \delta^2]$. What are the subgame perfect equilibria here?

Problem 5

Suppose the following game. There is a pile of N stones and two players. The players alternate taking 1, 2 or 3 stones from the pile. The person who removes the last stone loses and the other player wins. For what values of N does player 1 have a winning strategy and for what values of N does player 2 have a winning strategy? Prove your answer.