

Problem 1

Find all the pure strategy Nash equilibria of this game:

| | <i>H</i> | <i>D</i> |
|----------|----------|----------|
| <i>H</i> | 0, 0 | 1, 3 |
| <i>D</i> | 3, 1 | 2, 2 |

Problem 2

Remember rock, paper, scissors? Of course you do. Suppose that two players are playing for a dollar. If one person wins, the loser gives her a dollar. If they tie no money changes hands. Write the normal form of this game and find all the pure strategy Nash equilibria.

If you were playing with a friend who you thought was equally as smart and insightful as you are, how would you choose what to do?

Problem 3

What are the Nash equilibria of this game:

| | <i>L</i> | <i>R</i> |
|----------|----------|----------|
| <i>U</i> | 2, 2 | 0, 2 |
| <i>D</i> | 2, 0 | 1, 1 |

Problem 4

Find all the pure strategy Nash equilibria of this game:

| | <i>l</i> | <i>c</i> | <i>r</i> |
|----------|----------|----------|----------|
| <i>T</i> | 4, 4 | 3, 2 | 1, 2 |
| <i>M</i> | 1, 3 | 5, 3 | 6, 4 |
| <i>B</i> | 8, 1 | 8, 3 | 1, 2 |

Problem 5

Suppose two people are dividing a dollar. Each proposes an amount of the dollar she would like. If the two demands are compatible, i.e. they add up to no more than a dollar, then each individual gets what she demands. If, however, they add up to more than a dollar, neither gets anything. What are the Nash equilibria of this game? Do any of them strike you as particularly interesting?

Problem 6

A few moments ago we discussed the “Guess 2/3 of the average” game. Now, I’d like you to solve the “Guess the average game.” There are n players and each player guesses a real number in $[0, 1]$. The players who are closest to the average of all the guesses equally split a prize. What are all the pure strategy Nash equilibria for this game?

Problem 7

Suppose a game like the Stag Hunt, but with more players. Suppose that there are m hunters, but only n need to cooperate to catch the stag. (Assume $2 \leq n < m$.) If a stag is caught, the stag is divided equally among those who hunted stag. Like in the original game, a hunter who hunts hare gets a hare for sure which is worth 1.

Part A

Assume that the Stag is worth x where $x > m$. What are the Nash equilibria of this game?

Part B

What about the case where $m > x > n$?

Problem 8

Suppose two neighbors agree to improve a piece of shared property behind their houses. Each neighbor has a budget b of which she can spend any amount x_i on the shared property ($0 \leq x_i \leq b$). What is not spent on the shared property is spent on the individual’s own property. Assume each dollar spent improves the property by an equal amount, and each neighbor cares about the quality of the joint property slightly less than her own. In particular Suppose neighbor 1’s utility function looks like this:

$$\frac{3}{4}(x_1 + x_2) + (b - x_1)$$

And 2’s is similar:

$$\frac{3}{4}(x_1 + x_2) + (b - x_2)$$

What are the Nash equilibria of this game? Is this the best outcome of the game for each player?