Problem 1

Find all the pure strategy Nash equilibria of this game:

	l	c	r
T	$4,4$	$3,2$	$1,2$
M	$1,3$	$5,3$	$6,4$
B	$8,1$	$8,3$	$1,2$

Problem 2

Remember rock, paper, scissors? Of course you do. Suppose that two players are playing for a dollar. If one person wins, the loser gives her a dollar. If they tie no money changes hands. Write the normal form of this game and find all the pure strategy Nash equilibria.

If you were playing with a friend who you thought was equally as smart and insightful as you are, how would you choose what to do?

Problem 3

What are the pure strategy Nash equilibria of this game:

	L	R
U	2,2	0, 2
D	2,0	1, 1

Problem 4

Part A

Find all the pure strategy Nash equilibria for the following game:

	H	T
h	$3, 7$	$4, 4$
t	$4, 3$	$0, 5$

Part B

Find all the mixed strategy Nash equilibria.

Problem 5

Suppose two people are dividing a dollar. Each proposes an amount of the dollar she would like. If the two demands are compatible, i.e. they add up to no more than a dollar, then each individual gets what she demands. If, however, they add up to more than a dollar, neither gets anything. What are the pure strategy Nash equilibria of this game? Do any of them strike you as particularly interesting?

Problem 6

In Santa Fe there is a bar named El Farol. On a given night suppose that three groups of friends are considering going to El Farol (treat each group like it is a single player). El Farol is a cool bar, but it's a little too small to hold all three groups. If a group goes to El Farol and there are two or fewer groups there, the payoff from going is 2. If however all three go the payoff is -1/2. The utility for staying home for each group is 0.

What are the (pure and mixed) Nash equilibria in this game? Which equilibria are *socially optimal* – that is they maximize the sum of the utilities? Is there anything that makes the socially inferior Nash equilibria appealing?

Problem 7

Consider the following game which involves $n > 2$ players. Every individual chooses a number in the interval $[0, 1]$. Let μ represent the average of everyones' guesses. The person who is closest to $\frac{2\mu}{3}$ is declared the winner, and wins a prize worth some amount of money. If more than one person is equally close to this number, they split the prize equally. What are all the pure strategy Nash equilibria of this game? If you played the game against a random group of individuals would you guess your part of the Nash equilibria? Why or why not?

Problem 8

Suppose a game like the Stag Hunt, but with more players. Suppose that there are m hunters, but only n need to cooperate to catch the stag. (Assume $2 \leq n < m$.) If a stag is caught, the stag is divided equally among those who hunted stag. Like in the original game, a hunter who hunts hare gets a hare for sure which is worth 1.

Part A

Assume that the Stag is worth x where $x > m$. What are the Nash equilibria of this game?

Part B

What about the case where $m > x > n$?

Problem 9

Suppose two neighbors agree to improve a piece of shared property behind their houses. Each neighbor has a budget b of which she can spend any amount x_i on the shared property ($0 \leq x_i \leq b$). What is not spent on the shared property is spent on the individual's own property. Assume each dollar spent improves the property by an equal amount, and each neighbor cares about the quality of the joint property slightly less than her own. In particular Suppose neighbor 1's utility function looks like this:

$$\frac{3}{4}(x_1 + x_2) + (b - x_1)$$

And 2's is similar:

$$\frac{3}{4}(x_1 + x_2) + (b - x_2)$$

What are the Nash equilibria of this game? Is this the best outcome of the game for each player?