Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Part A

Suppose a choice set X and two relations $\succ$ and $\sim$ which satisfy all of the conditions listed in class. We will now define a new relation $\succsim$ the following way: $x \succsim y$ if and only if $x \succ y$ or $x \sim y$. Prove that $\succsim$ is transitive and complete.

Part B

Suppose a choice set X and a single relation $\succsim$ which is transitive and complete. Now we will define two new relations: $\succ$ and $\sim$.

- $x \succ y$ if and only if $x \succsim y$ and it is *not* the case that $y \succsim x$
- $x \sim y$ if and only if $x \succsim y$ and $y \succsim x$

Prove that $\succ$ and $\sim$ satisfy all the constraints specified in class

Problem 2

First a definition,

Definition 1. *A function $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing if and only if for all $x > y$, $f(x) > f(y)$ (informally, f assigns bigger outputs to bigger inputs).*

Suppose we have a choice set X which is finite and a preference relation $\succsim$ that is transitive and complete. We proved in class there there must be a utility function $u : X \to \mathbb{R}$ that represents $\succsim$. Prove that if $u(\cdot)$ represents $\succsim$ then for any strictly increasing function f, $f(u(\cdot))$ also represents $\succsim$.

Problem 3

Suppose that we have a choice function, $c : \mathcal{P}(X) \to \mathcal{P}(X)$, which satisfies Sen's α and β. Now I will define two relations from that function. First, $x \succsim^1 y$ if and only if $x \in c(\{x, y\})$. Second, $x \succsim^2 y$ if and only if there is some set S such that $x \in S$ and $y \in S$ and $x \in c(S)$.

Are $\succsim^1$ and $\succsim^2$ equivalent? If so, prove it. If not, illustrate a situation where they are not.

Problem 4

The book gives an axiom called *Houthakker's axiom.*

Definition 2. *If x and y are both in sets A and B and if $x \in c(A)$ and $y \in c(B)$, then $x \in c(B)$.*

I want you to prove three things

- Prove that if $c(\cdot)$ is a choice function and it satisfies Houthakker's axiom, then it must satisfy Sen's α.
- Prove that it must also satisfy Sen's β.
- Prove that if a choice function satisfies Sen's α and β then it satisfies Houthakker's axiom.

Graduate student problems (extra credit for undergrads)

Problem 5

Sen introduced another condition which is weaker than β, called γ,

Definition 3 (Sen's γ)**.** *If $x \in c(A)$ and $x \in c(B)$ then $x \in c(A \cup B)$.*

Show that Sen's α and γ is strictly weaker than Sen's α and β. (Do this by doing two things. First show that α plus β entails α plus γ, and then give an example of a choice function that satisfies α and γ, but not β.)

Problem 6

Suppose we have a choice set X which is composed of all pairs of real numbers between 0 and 1 (inclusive), $X = [0,1] \times [0,1]$. I illustrated a preference relation which cannot be captured by any utility function called a *lexicographic* preference relation.

- $(x_1, y_1) \succ (x_2, y_2)$ if $x_1 > x_2$
- $(x_1, y_1) \succ (x_2, y_2)$ if $x_1 = x_2$ and $y_1 > y_2$, and
- $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$ and $y_1 = y_2$

Show that this violates *continuity of* $\succ$ defined here,

Definition 4. *A relation $\succ$ on X is continuous if for all sequences $\{x_1, x_2, \ldots, x_n\}$ from X with limit x two things hold:*

- *if $x \succ y$ for some $y \in X$, then for all large n, $x_n \succ y$, and*
- *if $y \succ x$ for some $y \in X$, then for all large n, $y \succ x_n$.*