Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Choose one of the offered interpretations of probability and write 2-4 paragraphs (1-2 double spaced pages) criticizing that view as the unique correct view of probability. To remind you, the offered interpretations were: classical probability, propensity, objective relative frequency, hypothetical relative frequency, or subjective confidence (aka credence or subjective Bayesianism).

Problem 2

Part A

Construct a normal form decision with four options such that maximax, minimax, minimax regret, and maximize expected utility each suggest a different option. Illustrate why each decision rule chooses each action (be sure and provide a probability distribution over the states for the maximize expected utility decision rule).

Part B

Considering only the non-lexicographic versions of these decision rules, can any of them ever choose a strictly dominant option? What about a weakly dominant option? Give a brief explanation or example for each rule for both strict and weak dominance.

Problem 3

Recall the value of information problem in class. I had in my pocket two coins, one was fair ($P(\text{Heads}) = 0.5$) one was biased ($P(\text{Heads}) = 0.25$). I offered you the following gamble, if the coin came up heads I would pay you \$3, but if it came up tails you would pay me \$2. There are a total of six options. You could take the gamble immediately, you could refuse the gamble immediately, or you could choose to pay some amount a to observe a flip of the coin. If you chose to observe the flip, you saw whether the coin came up heads or tails and then you can take the gamble or refuse it – but in either case you had to pay me a for the information. (Do you see what the six options are?)

For what values of a does the action "Pay a, and then take the gamble if the coin comes up heads and refuse if tails" maximize expected utility compared to the others? What if instead of observing a test flip, I told you for sure whether the coin was fair or biased? What is the most you would be willing to pay then?

Problem 4

The following a simplified version of "Boole's inequality"

$$P(A \cup B) \leq P(A) + P(B)$$

Prove this is true for all A and B using only the axioms we discussed in class.

For extra credit, show this holds more generally:

$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i)$$

Graduate Student Problems (extra credit for undergrads)

Problem 5

I want you to illustrate a "counterexample" to Kreps theorem 3.7 by providing a preference relation which is not continuous (recall the definition from the last homework or from Kreps), but which nonetheless *can* be represented by a utility function. (I put "counterexample" in quotes, because I have left out a requirement from Kreps' theorem, the key to solving this problem is to figure out what.) Take the choice set to be $\mathbb{R}_+$ (the positive real numbers).

Problem 6

Here is a generalization of the fact from problem 3. Suppose a decision problem with n acts, $A_1, \ldots, A_n$, m states, $S_1, \ldots, S_m$, and you have a probability distribution over states given by $p(\cdot)$. The value of this decision problem, V, is just the expected utility of the action with highest expected utility.

Now suppose a partition on the states $Y = \{y_1, \cdots, y_r\}$ (where $r \leq m$ of course) and that I will tell you what partition we are in before you are required to choose an action. Prove that before I inform you of the partition, you now think this new decision problem has a value of at least V. Show that there are some cases where it might be greater than V.

This shows that the value of information is always greater than or equal to zero.

(HINT: Consistency requires that the probability you think I will say we are in y_x is given by $p(y_x)$.)