Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Construct a dutch book against someone who assigns the following probabilities:

$$\begin{aligned} P(A) &= 0.25 \\ P(B) &= 0.25 \\ P(A \cup B) &= 0.5 \\ P(A \cap B) &= 0.1 \end{aligned}$$

Show that in all circumstances the bookie who bets the way you suggest will make money.

Problem 2

Suppose that my utility function for money is given by $u(x) = ln(x+1)$. Construct a St. Petersburg lottery using the same coin example for this person which has infinite expected utility.

Problem 3

Recall the lexicographic preference ordering we discussed in class. I can choose two things: a proportion of the remaining macaroni salad ($x \in [0,1]$) and a proportion of the remaining mashed potatoes ($y \in [0,1]$). My choice set is made up of pairs (x, y) that represent the food I'm given. Suppose I choose first on the basis of macaroni salad, I want more rather than less. So $(x, y) \succ (w, z)$ if $x > w$. If the two options have the same amount of macaroni salad, only then do I compare the amount of mashed potatoes (again I want more). So $(x, y) \succ (w, z)$ if $x = w$ and $y > z$.

We can extend this to gambles as well. First I compare to gambles on the expected amount of macaroni salad (choosing the gamble which gives a higher expected amount of macaroni salad). Then if they are tied I choose on the basis of expected amount of potato salad.

This preference relation violates at least one of the von Neumann/Morgenstern axioms. Which one(s)? Show why it violates each one.

Problem 4

I want you to show that the axioms are "independent", that is no pair of them logically entails the third. I want you to give me three examples:

(a) Give an example of a preference relation that agrees with Axioms 1 and 2, but violates axiom 3. (b) Give another example that agrees with Axioms 1 and 3, but violates 2. (c) Finally give an example that agrees with Axioms 2 and 3, but violates 1.

(If applicable you can use examples given in class or in previous homeworks so long as you show why they fit each part.)

Graduate Student Problems (extra credit for undergrads)

Problem 5

Suppose the general setup for von Neumann/Morgenstern with a finite Z. Prove the following half of the representation theorem: If $\succ$ over Z can be represented by a utility function, then it obeys the three axioms of von Neumann/Morgenstern.

Problem 6

In problem 3 I gave you a preference relation which violates von Neumann/Morgenstern. Can you come up with an alternative axiomatization which preserves the general spirit of von Neumann/Morgenstern, but allows for preference relations of this type? We know that you cannot capture this preference relation with a single valued utility function, but can you come up with some other type of representation theorem? Give it your best shot.