Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Choose one of the offered interpretations of probability and write 2-4 paragraphs (1-2 double spaced pages) criticizing that view as the unique correct view of probability. To remind you, the offered interpretations were: propensity, objective relative frequency, hypothetical relative frequency, or subjective confidence (aka credence or subjective Bayesianism).

Problem 2

(*Borrowed from Ross' textbook*)

Part 1 A box contains three marbles: one red, one green, and one blue. Consider an experiment that involves taking one marble from the box then returning it to the box and taking a second marble from the box. What is the sample space? Suppose on each draw, each marble is equally likely what is the probability for each point in the sample space?

Part 2 What if the first marble is not returned to the box? What is the sample space now? What is the probability for each point?

Problem 3

Consider the following simple gambling game. There is a deck of cards that contains two types of cards "High" and "Low". You and "the dealer" are each dealt one card from this deck. You observe the card you have been dealt (but not the card the dealer has received), and are given the option of "surrendering". If you surrender, you give the dealer $50. If you do not surrender, you compare your card to the dealer's. If you have the high card and the dealer has the low, the dealer pays you $100. If you have the low and the dealer has the high you pay the dealer $100. If you both have the same card, nothing happens.

A strategy in this game contains two parts, "what to do if your card is high" and "what to do if your card is low." For instance the strategy (*stay*, *surrender*) would stay when you get the high card and surrender when you get the low card.

Part 1

There are four strategies in this game. Are any strictly dominated by another strategy? What about weakly dominated? If so, give the dominating strategy and show that it dominates the other.

Part 2

Suppose that the probability you get dealt the high card is p, suppose that the probability is exactly the same for the dealer as well. For all possible values of p, please identify the strategies which are judged best according to: (a) lexicographic minimax, (b) lexicographic maximax, (c) minimax regret, and (d) maximize expected utility.

Problem 4

Recall the value of information problem in class. I had in my pocket two coins, one was fair ($P(\text{Heads}) = 0.5$) one was biased ($P(\text{Heads}) = 0.25$). I offered you the following gamble, if the coin came up heads I would pay you \$3, but if it came up tails you would pay me \$2. There are a total of six options. You could take the gamble immediately, you could refuse the gamble immediately, or you could choose to pay some amount a to observe a flip of the coin. If you chose to observe the flip, you saw whether the coin came up heads or tails and then you can take the gamble or refuse it – but in either case you had to pay me a for the information. (Do you see what the six options are?)

For what values of a does the action "Pay a, and then take the gamble if the coin comes up heads and refuse if tails" maximize expected utility compared to the others? What if instead of observing a test flip, I told you for sure whether the coin was fair or biased? What is the most you would be willing to pay then?

Extra credit questions

Problem 5

The St. Petersburg lottery. Suppose you meet Jake who can (credibly) offer you the following gamble. Jake offers to flip a coin until it comes up head for the first time. He will then count the total number of flips and pay you \$$2^n$ where n is the total number of times he flipped the coin. If the coin came up heads on the first flip, you get \$2, if it came up *tails-heads* you get \$4, if it came up *tails-tails-heads* you get \$8, etc.

First, how much would you be willing to pay Jake for the opportunity to play this game? (Notice that you should be willing to pay at least \$2, since you are guaranteed to win at least that.)

Second, suppose that Shannon's utility function for money is the identity function (i.e. the utility of \$1 is 1, the utility of \$2 is 2, etc.). What is the expected utility of this gamble for Shannon? How much do you think Shannon should be willing to pay if she is an expected utility maximizer?

Do you and Shannon differ? If so, why do you prefer your answer?

Problem 6

Here is a general version of an earlier problem. Suppose a decision problem with n acts, $A_1, \ldots, A_n$, m states, $S_1, \ldots, S_m$, and you have a probability distribution over states given by $p(\cdot)$. The value of this decision problem, V, is just the expected utility of the action with highest expected utility.

Now suppose a partition on the states $B = \{b_1, \cdots, b_r\}$ (where $r \leq m$ of course) and that I will tell you what partition we are in before you are required to choose an action. Prove that before I inform you of the partition, you now think this new decision problem has a value of at least V. Show that there are some cases where it might be greater than V.

This shows that the value of information is always greater than or equal to zero.

(HINTS: (1) Consistency requires that the probability you think I will say we are in b_x is given by $p(b_x)$. (2) The expectation of an act can be represented as the weighted sum of the expectation of that act in each partition.)