Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

# Problem 1

For both parts let the choice set be $X = \{w, x, y, z\}$.

### Part 1

Suppose Jake has preference and indifference relations, $\succ$ and $\sim$, that satisfy the 9 conditions provided in class. Jake has the following preferences $w \sim x \succ y \succ z$. Jake's choices obey this preference relation – that is, when presented with a menu, he choses those objects that are most preferred from the menu. What will his choice function look like? (Specify what his choice function is for all 17 possible menus.) Convince yourself that his choice functions satisfies Sen's $\alpha$ and $\beta$.

### Part 2

Suppose Jake satisfies Sen's $\alpha$ and $\beta$, and he has a choice function that matches these two conditions:

- $c(\{w, x, y, z\}) = \{x, z\}$
- $c(\{w, y\}) = \{w\}$

Do you have enough information to give me a preference relation that will capture *all* of his choices for other menus? If not, say why not. If so, give me the preference relation that will do it.

# Problem 2

For this problem I would like you to give me three examples of a choice function. You must specify a choice set, $X$, and how the choice function behaves for all possible menus (all possible subsets of $X$). Please illustrate a choice function that:

1. Violates Sen's $\alpha$ but does not violate Sen's $\beta$
2. Violates Sen's $\beta$ but does not violate Sen's $\alpha$
3. Violates both Sen's $\alpha$ and $\beta$

(By giving the first two illustrations you have proven that Sen's $\alpha$ and $\beta$ are independent – that is that Sen's $\alpha$ doesn't entail $\beta$ or vice versa. Isn't that exciting?)

# Problem 3

Suppose a set of objects $O = \{w, x, y, z\}$ and a relation $R$ defined over the objects.

**Part 1**

I will tell you three things about $R$:

1. $xRy$
2. $R$ is symmetric
3. $R$ is transitive

Please choose one of the following: (a) $R$ is definitely reflexive, (b) $R$ is definitely irreflexive, (c) $R$ is definitely neither reflexive nor irreflexive, or (d) there isn't enough information to determine if $R$ is reflexive, irreflexive, or otherwise.

Justify your choice.

**Part 2**

I will tell you three things about $R$:

1. $xRy$
2. $R$ is reflexive
3. $R$ is transitive

Please choose one of the following: (a) $R$ is definitely symmetric, (b) $R$ is definitely asymmetric, (c) $R$ is definitely neither symmetric nor asymmetric, or (d) there isn't enough information to determine if $R$ is symmetric, asymmetric, or otherwise.

Justify your choice

# Problem 4

I mentioned in class that one of the nine conditions for preference relations is redundant. That is if we take this one mystery condition and remove it, we would **not** admit any additional preference relations as "rational". (Mathematically, we would say that one of the conditions is entailed by the other 8). Which one is redundant? Explain why you came to this conclusion (provide a proof of it using the other 8 if you can).

# Problem 5

Suppose an arbitrary choice set that includes at least three objects: $x, y$, and $z$. Suppose a preference relation, $\succ$, and an indifference relation, $\sim$, that satisfy the nine conditions in class. Can it be the case that $x \succ y$ and $x \sim z$ but not the case that $z \succ y$? If so, give a preference relation and a choice set where this happens. If not, give a proof of why it cannot.

HINT: A good way to start this problem is by assuming the nine conditions hold and that $x \succ y$, $x \sim z$ but that $z \succ y$ **does not** hold. Use the nine conditions to generate other constraints on what must hold between $x$, $y$, and $z$. If you end up generating a contradiction (like saying that it must be the case that $a \sim b$ and $a \succ b$), then you have proven that this is impossible. If you cannot generate a contradiction, then (probably) it's possible.

# Extra credit 1

Shannon is considering different colleges. She is planning to major in math and philosophy, and she's very dedicated to her education. Suppose her choice set is the set of all colleges to which she applied. Once she hears back from colleges, her "menu" will be the colleges that admitted her.

Shannon has decided on the following method for choosing where to go to school. When she find outs which colleges to which she has been admitted, she will whittle that choice set down to at most two options: the college that is best at philosophy and the one that is best at math. If those happen to be the same place, that's where she'll go. If she is put in a position to chose between a place that is better at math and another that is better at philosophy, she'll use the quality of the bars at that college as a tie breaker (but this consideration is only used in this case).

Depending on the choice set, Shannon might violate Sen's $\alpha$. Give me a choice set and the relevant choice functions that illustrate this violation.

# Extra credit 2

Suppose we have a choice set $X$ which is composed of all pairs of real numbers between 0 and 1 (inclusive), $X = [0, 1] \times [0, 1]$. I illustrated a preference relation which cannot be captured by any utility function called a *lexicographic* preference relation.

- $(x_1, y_1) \succ (x_2, y_2)$ if $x_1 > x_2$
- $(x_1, y_1) \succ (x_2, y_2)$ if $x_1 = x_2$ and $y_1 > y_2$, and
- $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$ and $y_1 = y_2$

Show that this violates *continuity of* $\succ$ defined here,

**Definition 1.** *A relation $\succ$ on $X$ is continuous if for all sequences $\{x_1, x_2, \ldots, x_n\}$ from $X$ with limit $x$ two things hold:*

- *if $x \succ y$ for some $y \in X$, then for all large $n$, $x_n \succ y$, and*
- *if $y \succ x$ for some $y \in X$, then for all large $n$, $y \succ x_n$.*