Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

In class I gave you the story about Lewd and Prude to illustrate Sen's paradox of the Parietian liberal. Give another example along the same lines that illustrates how minimal liberalism is inconsistent with Pareto.

Problem 2

The Condorcet cycle is a collection of three people with the following preferences:

$$\begin{array}{ccc} a \succ & b & \succ c \\ b \succ & c & \succ a \\ c \succ & a & \succ b \end{array}$$

This preference profile can generate problems for a variety of voting rules. I said in class that this cannot arise with "single peaked preferences" in a single dimension. For instance, if the candidates can be described as "more liberal" or "more conservative" than one another, and if the voters choose an optimal level of liberalness and vote for the candidates based on how close they are to idea, then the Condorcet profile cannot arise. What if the candidates are ranked not in one dimension but 2 – like "socially liberal/conservative" and "fiscally liberal/conservative." Can the Condorcet profile arise in this circumstance? Either say why not, or give an illustration of it arising.

Problem 3

Consider the following scenario for a candidate. There is one dimension along which she can locate a position and it will be represented by $[0, 10]$. She must locate her position on a whole number $0, 1, \ldots, 10$. Voters are uniformly distributed over the space, for simplicity assume there is a voter for every real number on the line. Voters will vote for whichever candidate is closest to them on the line. Suppose there are only two candidates and one has already chosen his position, x. Calculate what the optimal position for the other candidate is assuming that she wants to maximize her votes.

Suppose now that you are the first candidate and you know that the second will maximize her expected number of votes. What should you do?

Problem 4

The rich get richer Suppose Bill who has an unlimited betting amount meets a bookie who also has an unlimited betting amount. The bookie promises to offer Bill a gamble which has some probability $p > 0$ of resulting in a win for Bill. The bookie will continue offering Bill the gamble as long as Bill is willing to play, and he guarantees that each play is independent of the previous plays. Bill can choose how much to gamble at each stage. A win results in the bookie paying Bill the amount he bet, otherwise Bill pays the bookie that

Problem 4 continued on next page. . .

amount. Bill reasons that he can guarantee a win by adopting the following strategy. Bill will start with an amount \$$a$ that he would like to win. He bets \$$a$ on the first play and if he wins, he quits and walks away. If he loses he will bet \$$2a$ on the next game. If he wins he quits, and if he loses he plays again and bets \$$4a$. He will continue to double until he wins, but as soon as he wins he quits. Show that this strategy has an expected utility of \$$a$ regardless of p. Once you've done this, say why shouldn't I go to Vegas with this strategy.

Extra credit problems

Problem 5

Recall the formal setup for Arrow. There is a finite set of alternatives P (which has at least 3 members). Let R be the set of all preference relations on P (let's use $\succsim$ here) For some arbitrary N, let $f : R^N \rightarrow R$.

A few bits of notation. Let $a \succsim_{x_i} b$ mean that in profile $x \in R^N$, individual i prefers a to b. Let $a \succsim_{f(x)} b$ denote the statement f applied to preference profile x prefers a to b

Consider the following conditions on f (these are the standard Arrow conditions):

- **Unrestricted domain**. $f(x)$ is defined for all $x \in R^N$
- **No dictator**. There is no i such that for all profiles x, $f(x) = x_i$
- **Positive Association** For all $x \in R^N$ and $a, b \in P$, if $a \succsim_{f(x)} b$, then for all profiles x', $a \succsim_{f(x')} b$ so long as x' satisfies these two conditions
  - For all i such that $a \succsim_{x_i} b$, $a \succsim_{x'_i} b$
  - There is some j such that $a \not\succsim_{x_j} b$ but $a \succsim_{x'_j} b$
- **Citizen Sovereignty** For all $a, b \in P$, there is some x such that $a \succsim_{f(x)} b$
- **I.I.A.** For all $x \in R^N$ and all $a, b \in P$, if $a \succsim_{f(x)} b$, then for all x', $aS_{f(x')}b$ so long as x' satisfies these two conditions
  - For all i, $a \succsim_{x_i} b$ if and only if $a \succsim_{x'_i} b$
  - For all i, $b \succsim_{x_i} a$ if and only if $b \succsim_{x'_i} a$

I said in class that in light of the other conditions Positive Association + Citizen Sovereignty is equivalent to Pareto.

- **Pareto** If for all i $a \succsim_{x_i} b$ then $a \succsim_{f(x)} b$

Prove this.