Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Suppose Shannon assigns the following probabilities:

$$\begin{aligned} P(A) &= 0.25 \\ P(B) &= 0.25 \\ P(A \cup B) &= 0.5 \\ P(A \cap B) &= 0.1 \end{aligned}$$

Part 1

Construct a Dutch book against Shannon. Show that no matter what happens Shannon will lose money.

Part 2

Suppose that Shannon will get rewarded according to the reward scheme discussed in class. For each event E (here they are A, B, $A \cup B$, and $A \cap B$) Shannon will be charged money based on her "distance" from the truth. If she assigns event E probability x and E happens she must pay $-(1-x)^2$. If E doesn't happen she must pay $-(0-x)^2$. Give me a set of probability assignments that will do better than Shannon's *no matter the actual state of the world* (i.e. a set of probability assignments that will dominate the ones that Shannon has).

Problem 2

Consider Julie who is very afraid of taking gambles. Julie uses minimax to compare two different gambles. She starts with a preference relation $\succ$ over the prizes in X, and then compares two different lotteries using this preference relation. She looks at the worst prize (according to $\succ$) that has non-zero probability in each lottery and compares them. If one lottery, p, gives a better worst prize than another, q, she chooses p. If they are tied she looks at the second worst prize (according to $\succ$) – she uses lexicographic minimax.

Does Julie violate any of the axioms of von Neumann/Morgenstern? If so, which ones? Illustrate any violations that you might find.

Problem 3

I want you to show that the axioms are "independent," that is no pair of them logically entails the third. I want you to give me three examples:

(a) Give an example of a preference relation that agrees with Axioms 1 and 2, but violates axiom 3. (b) Give another example that agrees with Axioms 1 and 3, but violates 2. (c) Finally give an example that agrees with Axioms 2 and 3, but violates 1.

(If applicable you can use examples given in class or in previous homeworks so long as you show why they fit each part.)

Extra credit problems

Problem 4

Suppose we begin with a prize set that contains three beers $X = \{$ Pabst, Miller, Iron City $\}$. Jake assigns the following utilities to the three beers,

$$\begin{aligned} u(\text{Pabst}) &= 10 \\ u(\text{Miller}) &= 5 \\ u(\text{Iron City}) &= 0 \end{aligned}$$

If you give Jake two lotteries over different beers he will dutifully calculate the expected utilities of the lotteries and prefer the lottery with higher expected utility. But, there is one caveat: Jake's calculator rounds off to the nearest two decimal places. So if the expected utilities of two lotteries differ but round off to the same number, Jake will regard them as equivalent (and will therefore be indifferent). Will Jake violate any of the axioms of von Neumann/Morgenstern? If so, illustrate which ones and how.

Problem 5

Suppose the general setup for von Neumann/Morgenstern with a finite prize set X. Prove the following half of the representation theorem: If $\succ$ over X can be represented by a utility function, then it obeys the three axioms of von Neumann/Morgenstern.