Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

Problem 1

Remember Julie; she uses minimax to compare two different gambles. She starts with a preference relation $\succ$ over the prizes in X and a probability distribution over S. She compares two different actions using this preference relation and probability function. She looks at the worst prize (according to $\succ$) that has non-zero probability in each action and compares them. If one action, f, gives a better worst prize than another, g, she chooses f. If they are tied she looks at the second worst prize (according to $\succ$) – she uses lexicographic minimax.

Part A

Which axioms of Anscombe/Aumann does she violate?

Part B

Which axioms of Savage does she violate?

A note: If Julie violates Axiom 3 of Savage the preference relation "$\succ$ given a" really doesn't make sense. If it happens that she violates Axiom 3, consider the other axioms as if "$\succ$ given a" is replaced by simply "$\succ$".

Extra Credit

For extra credit say what axioms she would violate in each theory if she use non-lexicographic minimax.

Problem 2

I want you to give me a new example that makes state dependent utility seem as reasonable as possible. Without exactly duplicating the examples used in class please give me one example that conforms to Axioms 1-4 of Anscombe/Aumann but violates Axiom 5. Please describe in words the situation that makes it arise and defend it as at least moderately plausible.

Problem 3

Consider the state-dependent utility umbrella example in the context of Savage's theory. To put it more formally assume that $X = \{$ *umbrella*, *sunscreen* $\}$. Let S be infinitely large, but importantly consider two events $r \subset S$ is the event that it rains and $s \subset S$ is the event that it's sunny. Assume that Carlos has a probability distribution over S that assigns $P(r) > 0$ and $P(s) > 0$. Suppose he has the following utilities $u($ *umbrella when it rains* $) = 1$, $u($ *umbrella when it's sunny* $) = 0$, $u($ *sunscreen when it's sunny* $) = 1$, and $u($ *sunscreen when it rains* $) = 0$. Using this state-dependent utility function and his probability function, he maximizes his expected utility. What axioms of Savage will Carlos violate?

A note: If Carlos violates Axiom 3 of Savage the preference relation "$\succ$ given a" really doesn't make sense. If it happens that he violates Axiom 3, consider the other axioms as if "$\succ$ given a" is replaced by simply "$\succ$".

Extra credit

Problem 4

Prove that Savage's theory requires that one respect dominance by proving the following statement. Since this is used in some proofs of the representation theorem for Savage, prove this without appealing to that theorem.

Proposition 1. *Let $A = \{A_1, A_2, \ldots, A_n\}$ be any partition of S. Prove that if $f \succ g$ given A_i for all $A_i \in A$, then $f \succ g$.*

Problem 5

Suppose we begin with a prize set that contains three beers $X = \{$ Pabst, Miller, Iron City $\}$. Remember Jake from the last homework, he assigns the following utilities to the three beers,

$$
\begin{aligned}
u(\text{Pabst}) &= 10 \\
u(\text{Miller}) &= 5 \\
u(\text{Iron City}) &= 0
\end{aligned}
$$

Assume that Jake has a fully formed probability distribution over the states S. When comparing two actions Jake dutifully calculated the expected utility of that action using the his probability function and utility function. But, there is one caveat: Jake's calculator rounds off to the nearest two decimal places.

Part 1 Does Jake violate axiom 4 or 5 of Anscombe/Aumann?

Part 2 What axioms of Savage does Jake violate?