

Problem 1

For each of the sets of conditions listed below, give me an example of a “real world” relation which satisfies all the conditions described. Use one not mentioned in class, and briefly explain how it satisfies the conditions.

- (a) Transitive and symmetric
- (b) Reflexive, symmetric, but not transitive
- (c) Not symmetric, not reflexive, and not transitive

Problem 2

Suppose that we have four different foods: ice cream, chocolate sauce, french fries, and ketchup. These foods can be mixed together into a bowl in different proportions. Let (a, b, c, d) represent the amount of ice cream, chocolate sauce, french fries, and ketchup (respectively) that gets mixed together. For example, $(1, 1, 0, 0)$ represents 1 oz of ice cream mixed with 1 oz of chocolate sauce (and no french fries or ketchup). Or $(1, 0, 2, 0)$ represents 1 oz of ice cream mixed with 2 oz of french fries (and no chocolate sauce or ketchup).

Now, suppose a concatenation operation \oplus which works like this. (a, b, c, d) mixed with (w, x, y, z) produces a new mixture $(a + w, b + x, c + y, d + z)$. So, for example, $(1, 1, 0, 0) \oplus (1, 0, 2, 0)$ results in a food mixture consisting of 2 oz ice cream, 1 oz chocolate sauce, 2 oz french fries, and no ketchup.

Consider your run-of-the-mill preferences over all objects of the form (a, b, c, d) . Do they satisfy this axiom?

Definition 1 (Independence). *For all food bowls f and g , if $f \succ g$ then for any food bowl h , $f \oplus h \succ g \oplus h$*

Explain why or why not.

Problem 3

Suppose a set of objects $O = \{w, x, y, z\}$ and a relation R defined over the objects.

Part A

I will tell you three things about R :

1. wRx and yRz .
2. R is symmetric
3. R is transitive

Please choose one of the following: (a) R is definitely reflexive, (b) R is definitely irreflexive, (c) R is definitely neither reflexive nor irreflexive, or (d) there isn't enough information to determine if R is reflexive, irreflexive, or otherwise.

Justify your choice.

Part B

I will tell you three things about R :

1. wRx and yRz
2. R is reflexive
3. R is transitive

Please choose one of the following: (a) R is definitely symmetric, (b) R is definitely asymmetric, (c) R is definitely neither symmetric nor asymmetric, or (d) there isn't enough information to determine if R is symmetric, asymmetric, or otherwise.

Justify your choice

Graduate student questions (extra credit for undergrads)**Problem 4**

I mentioned in class that one of the nine conditions for preference relations is redundant. That is if we take this one mystery condition and remove it, we would **not** admit any additional preference relations as “rational”. (Mathematically, we would say that one of the conditions is entailed by the other 8). Which one is redundant? Prove it using the other axioms.

Problem 5

Can you come up with a different concatenation operation for the food bowls described in problem 2 that satisfies the independence axiom?