

Written answers are acceptable so long as they are legible. Remember, you can work with others but you must write the answers on your own. IF YOU WORK WITH OTHERS YOU MUST NOTE WITH WHOM YOU WORKED IN YOUR ANSWER.

**A note:** If Jackie, Dean, or Carlos violates Axiom 3 of Savage the preference relation “ $\succ$  given  $a$ ” really doesn’t make sense. If it happens that any of them violates Axiom 3, consider those axioms where “ $\succ$  given  $a$ ” is replaced by simply “ $\succ$ .”

## Problem 1

Consider the following four acts:

	<i>Rain</i>	<i>Sun</i>
$A_1$	Sunscreen	Sunscreen
$A_2$	Umbrella	Sunscreen
$A_3$	Sunscreen	Umbrella
$A_4$	Umbrella	Umbrella

Suppose that Jackie has the following preferences:  $A_2 \succ A_1 \succ A_4 \succ A_3$ . Which axioms of Savage is she guaranteed to violate? Illustrate the violations that you find.

## Problem 2

Consider Dean. For Dean there are two prizes  $Z = \{\$0, \$1\}$ . Dean has a subjective probability function over the states, and he groups all actions into two types: those that give him an expected monetary gain of \$0.50 or higher, and those that do not. When comparing two actions Dean uses this simple rule for forming his preference. If one act gives him a \$0.50 or higher expected monetary gain and the other does not, he prefers the former. Otherwise he’s indifference.

What axioms of Savage does Dean violate? Illustrate the violations that you find.

## Problem 3

Remember Carlos, who had a prize set with three soccer jerseys  $Z = \{ \text{Barcelona, Manchester United, FC Bayern} \}$ . Carlos assigns the following utilities to the three jerseys,

$$\begin{aligned} u(\text{Barcelona}) &= 7 \\ u(\text{Manchester United}) &= 3 \\ u(\text{FC Bayern}) &= 0 \end{aligned}$$

Carlos also has a probability distribution over a set of states of the world,  $S$ . If you give Carlos two acts he will dutifully calculate the expected utilities of the acts and prefer the act with higher expected utility. But, there is one caveat: Carlos' calculator rounds off to the nearest two decimal places. So if the expected utilities of two acts differ but round off to the same number, Carlos will regard them as equivalent (and will therefore be indifferent).

Which axioms of Savage does Carlos violate? Illustrate the violations that you find.

## Graduate student problems (extra credit for undergrads)

### Problem 4

Prove that Savage's theory requires that one respect dominance by proving the following statement. Since this is used in some proofs of the representation theorem for Savage, prove this without appealing to that theorem.

**Proposition 1.** *Let  $A = \{A_1, A_2, \dots, A_n\}$  be any partition of  $S$ . Prove that if  $f \succ g$  given  $A_i$  for all  $A_i \in A$ , then  $f \succ g$ .*

Also prove the weaker version:

**Proposition 2.** *Let  $A = \{A_1, A_2, \dots, A_n\}$  be any partition of  $S$ . Prove that if  $f \succsim g$  given  $A_i$  for all  $A_i \in A$ , then  $f \succsim g$ .*

### Problem 5

Recall the definition of "more probable than" that I gave in class.

**Definition 1.** *For all events  $a, b \subset S$   $a \succ^* b$  if and only if there exists an  $x, y \in Z$  such that,  $x \succ y$ ,  $f(a) = x$ ,  $f(-a) = y$ ,  $g(b) = x$ , and  $g(-b) = y$  and  $f \succ g$ .*

Prove that if Kim satisfies Savage's axioms, then it cannot be the case that  $a \succ^* b$  and  $b \succ^* a$ .